Probabilistic Evolutionary Process: a possible solution to the problem of time in quantum cosmology and creation from nothing

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March 11, 2009

Abstract

We present a method, which we shall call the probabilistic evolutionary process, based on the probabilistic nature of quantum theory to offer a possible solution to the problem of time in quantum cosmology. It offers an alternative for perceiving an arrow of time which is compatible with the thermodynamical arrow of time and makes a new interpretation of the FRW universe in vacua which is consistent with creation of a de Sitter space-time from nothing. This is a completely quantum result with no correspondence in classical cosmology.
PACS: 98.80.Qc
Keywords: Quantum Cosmology.

1 Introduction

In this work we show that the probabilistic nature of quantum mechanics can play an essential role to suggest a possible solution to the problem of time in quantum cosmology. We have given the acronym "Probabilistic Evolutionary Process" (PEP) to this method. In addition to the problem of time mentioned above, PEP can account for the arrow of time too, which is compatible with thermodynamical arrow of time. PEP also opens a new window to study quantum cosmological scenarios such as the "creation from nothing" scenario which has been the focus of attention over the past decades. Let us then start by focusing on the problem of time.

1.1 The problem of time

One of the intriguing notions in theoretical physics is the meaning of time which plays a crucial role in classical as well as quantum physics. Over the past decades, a huge amount of effort has been concentrated on defining exactly what one means by time. In Newtonian mechanics, time is an external parameter upon which the evolution of other independent parameters depends. So to describe the evolution of a system, the parameters of the system are written in terms of time. However, as we now know, Newtonian mechanics is only correct when the speeds involve are small compared to the speed of light or the gravitational fields are weak. We also know that the fundamental theory is General Relativity (GR). In GR, time is a coordinate like the others. General relativity is based on the principle of general covariance which basically states that all observers must see the same physics. This principle causes GR to become a gauge theory or, as is required of such theory, invariant under diffeomorphism transformations. The diffeomorphism invariance suppresses any manifestation of time

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in the quantum version of such theories. This absence of time in diffeomorphism invariant theories is known as the problem of time, meaning that there is no evolution in such theories, for a comprehensive review see [1]. Not surprisingly, to address this problem in GR and cosmology, a considerable amount of work has been done over many years [1]. In this work we shall offer a new prescription as an alternative which may address the problem of time in such theories. This method is based on the probabilistic nature of the wave functions in quantum mechanics.

The notion of time suffers from another problem as well and that is the problem of the direction of evolution [2]. As is well known, all the fundamental theories are invariant under time reversal, namely, $t \to -t$. This of course means that going along a specific direction in time is the same as going along the opposite direction. However, in retrospect, nature seems to take a preferred direction for time. For example, in thermodynamics the arrow of time is naturally defined by the second law of thermodynamics which states that entropy can only increase. The same is true in observational cosmology in which observations show that the universe is expanding[1]. A more tangible example is the psychological arrow of time, that is the ordinary sense of time in every day life when we make a distinction between the past and future. Among the above we concentrate on thermodynamical and cosmological arrows of time. An important question to ask then is: are these different arrows of time compatible with each other [2]? We will argue that using PEP, one can accommodate an arrow of time which is compatible with the fundamental thermodynamical concepts.

1.2 Creation from nothing

It would be very interesting if the present universe with all its immense complications could be shown to have evolved from the simplest of initial states, namely the vacuum. This is the basic motivation behind the efforts of those who believe in a unified theory of everything. The concept of a unified theory led to the construction of the electro-weak theory and later to the standard model in particle physics which accounts for the strong interactions as well. Nevertheless, the intractable gravitational force is still stubbornly difficult to accommodate into this scheme. A huge amount of work has been done to combine gravitational forces with others, but to no avail. The most promising example is the string theory which has made noticeable strides towards this goal, but is still far from having a firm ground to stand on.

One of the approaches to unification is the geometrization of matter fields. This means that the matter fields are attributed to and emerge from the geometry of the space-time. The Kaluza-Klein model [3] belongs to this category where the appearance of an extra dimension can play the role of the electromagnetic four vector potential. On the other hand, in quantum field theory in curved space-time, it is shown that the expansion of the universe leads to the creation of matter particles [4]. In these models it is often the case that the process of expansion is introduced by hand and is therefore artificially woven into the fabrics on which the model is based without any fundamental reason as to the existence of such processes. Recently, Ambjorn et al. [5] have shown that the present accelerating phase of the universe could have resulted from quantum dynamical gravity as a result of the causal dynamical triangulation (CDT).

2 Probabilistic Evolutionary Process

To quantize a classical model, the following procedure is commonly followed. The classical Hamiltonian is written in its corresponding operator form where, upon quantization, a Schrödinger type equation, i.e. $i\hbar\frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi$, becomes the prevalent dynamical equation from which the time evolution of the quantum states may be ascertained. However, in diffeomorphism invariant models the Hamiltonian becomes a constraint, that is, $\mathcal{H} = 0$. These models cannot then provide for the evolution of the corresponding states and this means that in these models all the states are stationary. Well

[1]At least we are presently in the era of an expanding universe.

known examples of such models are GR and cosmology. In quantum cosmology, Schrödinger equation becomes the Wheeler-deWitt equation (WD), $\mathcal{H}\Psi = 0$. The problem is then arises as to how one can describe the evolution of the universe, since the universe is not in a stationary mode as far as the present observational data suggests. To provide an answer to the question of time, different proposals have been introduced in different forms, ranging from their implementation before or after quantization or discarding time altogether [1]. However, it seems as if none of these mechanisms work for simple models such as the FRW metric in vacua. Since almost all of these proposals introduce a parameter (field) to represent time, they need at least another parameter to describe its evolution. In other approaches, one considers the Hamiltonian constraint itself to find a relation between the parameters (fields), since one has $\mathcal{H}(a, b) = 0$. Therefore, one can write one parameter in terms of the other, e.g. $a(b)$ and $b(a)$, and interprets them as the relational behavior of different fields. These two examples show that the above mechanisms cannot work for models with only one free parameter.

In a previous work [6] we introduced a mechanism based on the probabilistic structure of quantum systems that can accommodate systems with only one degree of freedom. In quantum systems the square or the norm of a state represents the probability, that is, $\mathcal{P}_a = |\Psi(a)|^2$. Now, PEP suggests that the state Ψ_a makes a transition to the state Ψ_{a+da} if their distance, da, is infinitesimal and continuous[2]. The probability of transition[3] is higher if $\mathcal{P}_{a+da} - \mathcal{P}_a$ is larger[4]. The mechanism for transition from one state to another is based on utilizing the effects of a small perturbation[5,6,7]. To grasp the features of the concept of PEP described above in a more clear fashion, we resort to a simple example given in figure 1 which has been borrowed from the companion paper [8]. Note that the figure is for a simple model with one degree of freedom (a scale factor). For a more general model the horizontal axis should be interpreted as the whole degrees of freedom of the universe such as scale factors, matter fields and so on.

Let the initial condition be, for example, $a = 2.5$, the point P in figure 1. Then PEP states that the system (here the scale factor or a) moves infinitesimally close towards a state with higher probability and consequently P moves to the right to reach Q, a local maximum and, therefore, stays at Q. It means that the scale factor remains constant as the time passes[8]. We denote this transition by $PEP : P \rightarrow Q$ or equivalently $P \overset{PEP}{\longrightarrow} Q$. Now let the initial condition be the point R. Then we have $R \overset{PEP}{\longrightarrow} Q$, and so on. Note that $R \overset{PEP}{\longrightarrow} S$ is possible but it has much smaller probability due to $R \overset{PEP}{\longrightarrow} Q$. We also note that the transitions $R \overset{PEP}{\longrightarrow} S$ and $S \overset{PEP}{\longrightarrow} T$ can be interpreted as a tunnelling precess in ordinary quantum mechanics. It means that PEP can reproduce a tunnelling process but with a very small probability of occurrence.

The power of addressing time in this fashion is that it is based on the probabilistic nature of quantum mechanics which is inherent in the wave function and is deduced here from the WD equation, hence the notion of time is built on the concept of probability. In comparison with other models, PEP does not need any extra treatment like reparameterization by another field or addition of other

[2] More precisely, the initial state is around Ψ_a since the $\mathcal{P}_a = |\Psi(a)|^2$ is the probability density.

[3] This transition probability can play the role of the speed of transition, i.e. the higher the probability of transition the larger the speed of transition.

[4] Since there is no constraint on the positivity of $\mathcal{P}_{a+da} - \mathcal{P}_a$, PEP can then describe tunnelling processes too. This feature of PEP was not investigated in [6].

[5] Certainly, in quantum cosmology, the universe is considered as one whole [1, 7] and the introduction of an external force is irrelevant. However, because of the lack of a full theory to describe the universe, these small external forces are merely used to afford a better understanding of the discussions presented here.

[6] This perturbation can be caused by the fact that, for example, the scale factor operator, A, does not commute with the Hamiltonian, $\mathcal{H}$, $[A, \mathcal{H}] \neq 0$. This physically means that when we restrict the wave function to an eigenfunction of the Hamiltonian, $\mathcal{H}\Psi(a) = 0$, it cannot be an eigenfunction of A simultaneously i.e. $A\Psi(a) \neq a\Psi(a)$. So the initial condition $a = a_0$ is not a steady state and hence it cannot be at a_0 and must move away from its initial value. The rule for such moves are given by PEP.

[7] The origin of the perturbations may be rooted in the von Neumann approach to quantum mechanics. The perturbation could naturally arise from the fact that in quantum mechanics one does not know the position of a state precisely.

[8] A perturbation around local maximum is acceptable as mentioned before.

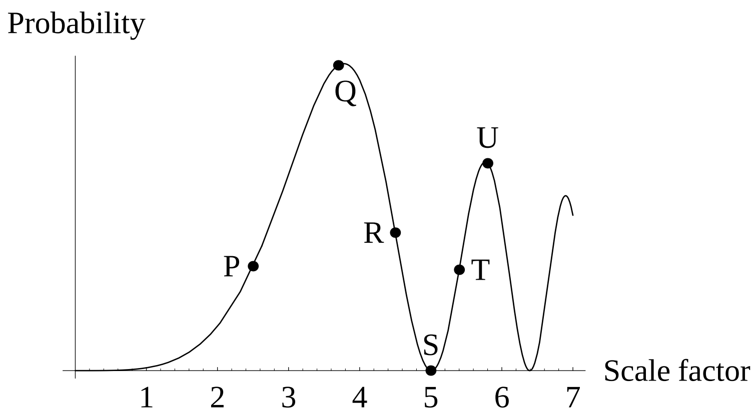

Figure 1: Figure used to explain the idea of PEP.

fields to realize the desired behavior.

As an evidence for PEP, we have shown that canonical quantization of cosmology (WD equation) together with PEP is physically equivalent to the deformed phase-space quantization of cosmology, at least for the models discusses in [8]. For more thorough understanding of the discussion at hand, we note that along with the canonical and path integral approaches to quantization, another procedure also exists, the so-called deformed phase-space [9]. This approach is well known and much work has been done in this regard [8, 10]. In this kind of quantization, the additional terms emanating from the deformation of phase space modify the classical Hamiltonian. These extra terms can either be interpreted as quantum effects [9, 10] or as a quantum potential in the Bohmian version of quantum mechanics. An important question for these models is how does the deformed phase-space quantum cosmology relates to canonical quantum cosmology or the path integral approach to quantum cosmology? In [8] it is shown that based on PEP, the canonical approach and phase-space deformed approach are physically equivalent, at least for the models currently under discussion.

3 The physics behind PEP

The basis of PEP are on the probabilistic interpretation of quantum mechanics and more specifically on the probabilistic interpretation of $|\Psi|^2$, where Ψ is the wave function of the system under consideration. In a few words, PEP makes a duality between "growing in time" and "increasing in probability", where probability is read from the corresponding $|\Psi|^2$. To make a convenient description of PEP, we must insist on a special feature of quantum cosmology which is nothing but the fact that our system is the universe alone. To be more clear, we adopt the following two different viewpoints for describing the universe.

In the first viewpoint, there is an external observer who is outside the system under discussion. To describe the thermodynamical properties of that system the observer needs a large number of copies of the system. These copies can be made by either of the two following approaches: the observer can make a large number of copies and observes them simultaneously or the observer has only one copy of the system and observes it over a long period of time. These two approaches are standard and equivalent in thermodynamics when describing a system. Here, the observer can see all the possibilities with the appropriate weights and therefore can, for example, normalize the results of the observations to 1, and may thus predict the future of the system.

In the second viewpoint, the observer is basically the system itself and can only change its initial conditions as long as it remains close to the initial state of the system. The observer cannot see all the possibilities in the same way as the observer mentioned in the first viewpoint could, but can only see, as it were, himself and his neighbors. Now the notion of normalization to 1 is not necessary and even

relevant since in this viewpoint only the relation between the initial point and its neighbors becomes important. Therefore, here the notion of probability could be replaced by a more meaningful one, namely the possibility.

The first viewpoint discussed above is the commonly used method for describing the behavior of a system since, as an observer outside the system, we can produce as many copies of the system as we want and calculate all the appropriate averages. However, this is not the case when the system under consideration is the universe itself. There is only one copy of this system and, more importantly, the observer is internal to the system and not external. Therefore, mathematically, in assigning $|\Psi|^2$ to the universe we do not need to normalize the wave function since as mentioned above, the universe or equivalently the observer can only see its neighbors. Therefore, it is more convenient to ignore the notion of probability and change it to possibility in describing the second viewpoint. It is worth mentioning that in general, as is well known, because of the inner product problem in quantum cosmology, the notion of probability is not well defined. Such a notion however, becomes redundant in PEP and is replaced by the notion of possibility which, in spite of the inner product problem, is well defined in the present context and can be used unambiguously.

To make the discussion more clear, consider a particle moving under the influence of a potential of a certain field. From the point of view of the particle, or an observer moving with the particle, it can, in principle, move infinitesimally close to any of its physically allowed neighboring points, but it chooses a point with a lower potential since it experiences the force $\vec{F} \propto -\nabla V$, where $\vec{F}$ and V are the force and its corresponding potential respectively. In summary, a standard particle moves without any prior knowledge of the properties of the far points (points except those that are in its neighborhood) and as a result, the particle would end up in a local minimum of the potential and not necessarily in a global minimum. This is much the same as the behavior of the universe taken as the system, discussed above. In quantum cosmology, the degrees of freedom of the minisuperspace play the role of position in the above example and the function $|\Psi|^2$ plays the role of the potential. We shall present an extended discussion on PEP in the Conclusions section.

4 Thermodynamical arrow of time and PEP

The root of thermodynamical arrow of time is in the second law of thermodynamics (SLT), stating that “entropy is not decreasing”, or, putting in mathematical form $\triangle S \geq 0$, where S is the entropy of a closed system. Note that the entropy of a system, like its energy, becomes meaningful only if compared to a defined standard or another system. The second law of thermodynamics opens the door to an important physical controversy, namely time reversal since any macroscopic system would evolve to a more disordered state, starting from an initially ordered sate. To account for this transition one can count all the possible micro states of the system and calculate the corresponding entropy as

$$S = k_B \log N \tag{1}$$

where k_B is the Boltzman constant and N is the number of all the possible different microstates for a definite macrostate. Explicitly, it means that for a given macroscopic system with a finite number of degrees of freedom, finite volume and finite temperature, the number of allowable microstates is N. This shows the high degree of correlation between thermodynamics and combinatorial arithmetics of micro-structures. Here the similarity of SLT and PEP becomes clear since they have a common base, the microstates possibilities.

This monotonic behavior of entropy is very convenient for a definition of time. It means that time is a monotonic function of the entropy which can mathematically be stated as $\triangle t = t_f - t_i = f(\triangle S = S_f - S_i)$ in which f is a monotonically increasing function and S_f and S_i are the entropy of the final and initial states at times t_f and t_i respectively. Note that the function f is free of any constraint except the monotonically increasing behavior and so its form is unknown at least up to the uncertainty in our knowledge at present.

An important problem here is that when we speak of the universe as a whole what becomes of the meaning of microstates? We have only one macrostate, the universe! Here, we just assume a duality between PEP and SLT due to their common sensitive responses to possibilities. One suggestion is that we can interpret isotropicity and homogeneity constraints in quantum cosmology, as macroscopic constraints[9]. So we can interpret the constraints as defining the macroscopic states i.e. the macroscopic structure is fixed by the constraints like temperature, pressure and the number of particles in the thermodynamical system which would define the macroscopic structure of the system. The resulting wave function that satisfies the macroscopic constraint then shows the possibilities of microstates which satisfy the macroscopic constraints and also the weight of any one of these possibilities with respect to the others. In this view the relation between PEP and SLT becomes more clear.

5 Creation from nothing

Here we study a simple FRW model to show how PEP can predict non-trivial (non-vacuum) solutions from trivial (vacuum) ones. Let us take the FRW metric with zero curvature

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{2}$$

where $N(t)$ and $a(t)$ are the lapse function and scale factor respectively. The corresponding action becomes

$$\begin{aligned}\mathcal{L} &= \sqrt{-g}(R[g] - V(a)) \\ &= -6N^{-1}a\dot{a}^2 - Na^3V(a),\end{aligned} \tag{3}$$

where $V(a)$ is related to an arbitrary matter field and the total derivative terms are ignored in the second line. The corresponding Hamiltonian becomes

$$\mathcal{H}_0 = \frac{1}{24}Na^{-1}p_a^2 - Na^3V(a). \tag{4}$$

Since the momentum conjugate to $N(t)$ does not appear in the above Hamiltonian it is a primary constraint. Therefore, to obtain the full equations of motion we shall work with the Dirac Hamiltonian which is more appropriate

$$\mathcal{H} = \frac{1}{24}Na^{-1}p_a^2 - Na^3V(a) + \lambda\pi, \tag{5}$$

where π is the momentum conjugate to $N(t)$ which is added through a Lagrange multiplier, λ, as a primary constraint. The corresponding equations of motion are

$$\begin{aligned}\dot{a} &= \{a, \mathcal{H}\} = \frac{1}{12}Na^{-1}p_a, \\ \dot{p_a} &= \{p_a, \mathcal{H}\} = \frac{1}{24}Na^{-2}p_a^2 + 3Na^2V(a) + Na^3V'(a), \\ \dot{N} &= \{N, \mathcal{H}\} = \lambda, \\ \dot{\pi} &= \{\pi, \mathcal{H}\} = -\frac{1}{24}a^{-1}p_a^2 + a^3V(a),\end{aligned} \tag{6}$$

where a prime represents differentiation with respect to the argument. To preserve the primary constraint, $\pi = 0$, at all times the secondary constraint must be satisfied naturally i.e. $\dot{\pi} = 0$. Due to

[9] Also, weaker or stronger constraints. This viewpoint is not all that strange since really the isotropicity and homogeneity are macroscopic symmetries but are broken in microscopic regimes, e.g. in the Milky way.

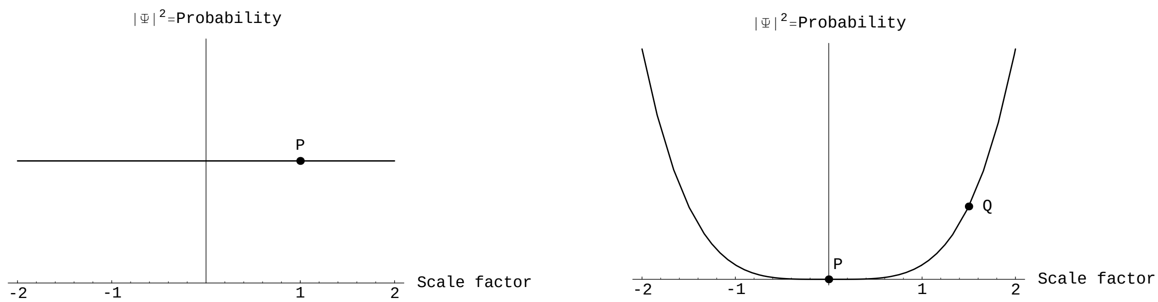

Figure 2: The left figure shows the trivial case and the right one the non trivial (quantum) case.

the latter constraint and the above equations, the equation of motion in the comoving gauge $N(t) = 1$ becomes

$$\dot{a} = \sqrt{\frac{1}{6}a^2V(a)}, \tag{7}$$

which is the familiar Friedman equation. The above equation has been solved for different kinds of matter which are represented by $V(a)$, such as radiation, dust and the cosmological constant. It is obvious that the above equation for vacuum, $V(a) = 0$, reduces to the trivial Minkowskian metric. It means that classical general relativity predicts only the trivial solution for an isotropic and homogeneous space-time.

Now let us study the above simple model in the quantum regime. To quantize the model we follow the Dirac approach to get the Wheeler-DeWitt equation as $\hat{\mathcal{H}}\Psi(a) = 0$ which for our model is

$$\hat{\mathcal{H}}\Psi(a) = \left[\frac{1}{24}p_a a^{-1} p_a - a^3 V(a)\right]\Psi(a) = 0, \tag{8}$$

where a certain ordering is assumed and $[a, p_a] = i$ ($\hbar = 1$). In the a-representation, the above equation transforms to the following differential equation

$$\partial_a^2\Psi(a) - a^{-1}\partial_a\Psi(a) + 24a^4V(a)\Psi(a) = 0. \tag{9}$$

To compare quantum solutions with the classical ones, we restrict ourselves to the vacuum case $V(a) = 0$. The solution becomes

$$\Psi(a) \propto \begin{cases} c_1, \\ \\ c_2 a^2, \end{cases} \tag{10}$$

where c_1 and c_2 are integration constants. Here we make our interpretation using PEP to describe the above solutions.

5.1 First case

For the first solution if one chooses an initial condition for the scale factor, it remains in this initial condition since the norm of the scale factor is a constant i.e. $P\xrightarrow{PEP}P$, as the left plot in figure 2 shows. It means that the scale factor is a constant which is the trivial Minkowskian solution similar to the classical solution.

5.2 Second case

This case is our main result and has no counterpart in the classical case, noting that all the results here are of quantum nature. The norm of the scale factor predicts an unfinished expansion for the

scale factor due to PEP i.e. $P \stackrel{PEP}{\longrightarrow} Q$, the right plot in figure 2. Note that this behavior does not have a classical counterpart. Physically, it means that quantum effects cause an expansion even for vacuum. This prediction is important for the unification of all forces. It also means that if the initial state is the vacuum, the quantum effects cause the expansion and the expansion in turn creates particles. In summary, the present state of the universe with all matter fields is a result of the vacuum state. This is only a result of the dynamical interpretation of quantum cosmological solutions by PEP.

Such interpretation of the wave function makes the usual WD equation comparable with other, more complicated models like the Causal Dynamical Triangulation method (CDT) for quantizing general relativity. In [5] it is shown that the present de Sitter phase of the universe can be reached from a vacuum initial condition due to the evolution rules laid down by CDT. If one believes in the results of [5], some approximation can then be presented using the PEP. For example, in [5] the resulting universe has an exponential behavior in time, so a rough calculation shows, using (10)

$$\begin{cases} a(t) = e^{\sqrt{\frac{\Lambda}{3}}t} \\ \\ p = c_2^2 a^4 \end{cases} \qquad \Rightarrow \qquad \text{time} = \tfrac{1}{L}\log(p/p_0), \tag{11}$$

where $L = \sqrt{\frac{3}{16\Lambda}}$ and p_0 is an arbitrary constant. Note that in the PEP viewpoint we do not have any cosmological constant, and therefore to make it consistent we must rewrite the multiplier with an appropriate constant which is defined in the model. Since in quantum cosmology c, G and $\hbar$ are defined, one must write L as a function of these constants in a such a way that L has the dimension of 1/time. So it is natural to choose $L = t_p^{-1}$ and therefore[10]

$$t = \sqrt{\frac{\hbar G}{c^5}} \log p/p_0. \tag{12}$$

Is the above result exact? Not really, even if we believe that the quantum vacuum will lead to an exponential behavior for the universe. Since in the present epoch the behavior of the scale factor is believed to be of a power law type, having a different type for earlier times, it then makes sense to regard it as possible that the form of the relation between time and probability must be changed at least for small scale factors.

Note that as was mentioned, the above results are direct consequences of quantum geometry. In summary, the vacuum state of quantum geometry may lead to a non-vacuum state in a classical framework. This result is the goal of all physicists who pursue the notion of unification.

Now, suppose the initial scale factor is zero, $a(0) = 0$, point P in figure 2. In this case the 3-geometry becomes the 0-geometry and is in an unstable equilibrium. The universe exits from this point due to PEP and the above discussions become relevant. The point here is that one can make a similarity between the initial 0-geometry, point P, and the Vilenkin's creation from nothing such that "nothing refers to the absence of not only matter but also space and time" [11]. We note that in our simple model, for the initial point P we have neither space, since it is the 0-geometry, nor matter, as assumed by imposing the condition $V(a) = 0$.

6 Conclusions

We have introduced a method to interpret the evolution of the wave function of the universe using the probabilistic evolutionary process (PEP). PEP is based on the probabilistic interpretation of quantum mechanics. The PEP's rule is that the universe can evolve to a state in its neighborhood if the latter is more probable. We have shown that this kind of interpretation of the wave function, in addition to suggesting a possible solution to the problem of time in quantum cosmology, makes a definition

[10]It would be interesting to observe that if one believes in the relation between entropy (1) and time (11), then a relation between the Boltzmann constant and Planck scale becomes natural.

of an arrow of time possible. The PEP's arrow of time coincides with the thermodynamical one due to the representation of the latter by microstate possibilities (probabilities). In a companion paper [8] we have shown that the prediction of canonical quantum cosmology with PEP is equivalent to deformed phase space quantum cosmology. This feature can be interpreted as an evidence for PEP even if this correspondence is true only for some models. Finally, we have shown that PEP predicts a nontrivial (e.g. de Sitter) solution from a trivial (vacuum) quantum state, that is, creation from nothing. One may extend this method to more complicated models, but even an example as simple as the one presented in this paper results in interesting and non trivial features, namely a possible resolution of question of time in quantum cosmology and a mechanism for creation from nothing. Let us present a quote from [5] which is particularly relevant to our discussion here, "to show that the physical space time surrounding us can be derived from some fundamental, quantum-dynamical principle is, a holy grail of theoretical physics".

Since the PEP is in its first steps of development, there naturally arises some questions, e.g. what are the equations governing the dynamics of transition from a low probability states to a higher one, or what is the correspondence between PEP and semi-classical wave functions etc. As for the first question, since such a transition is related to a change in the entropy, the natural choice to describe the dynamics of the transition resides in the dynamics of the increasing entropy in non-equilibrium statistical mechanics. Now, it is commonly known that the evolution of the entropy depends on the microscopic structure of the macroscopic system under consideration. As was mentioned above, we may imagine the scale factor as being a macroscopic quantity so that its evolution would depend on the microscopic degrees of freedom of the system, namely the universe. The coarse graining structure of the space time considered in the literature [12] is an important example relevant to the present discussion. Any further discussion relating to this matter should naturally await the emergence of a full quantum theory of gravity. As far as the second question is concerned, namely the semi-classical wave function, we will see that the approach is not relevant to our example and results. The first step in establishing the classical-quantum correspondence is the decomposition of the wave function according to, $\Psi = Re^{iS}$. It has been mentioned that in this approach if $S = 0$ then one cannot work in an appropriate manner since the method breaks down [13]. This is exactly what we encounter here since in our toy model the absence of any potential term in the Lagrangian causes the vanishing of S. This poses no contradiction to our interpretation of the second equation in (10) since we interpret it as a pure quantum result without any counterpart in classical cosmology.

Finally, an interesting point to note is that the time variable in the previous sections (specially in section 5) is a coordinate (gauge) variable whereas the entropy seems to be a quantity which is independent of the observer. The question then arises as to how such a gauge dependent variable, that is time, can be related to entropy. This should cause no alarm here since the relation between the time coordinate and number of possibilities has some roots in the notion of entropy and SLT. In section 4, we introduced a function f which establishes the correspondence between time and entropy in such a way as to make the former a monotonically increasing function of the latter without any further constraint. However, the above discussion results in an additional constraint on this function. Since such a function relates the number of possibilities (entropy) to time or, a gauge independent quantity to a gauge dependent quantity, it has to be a gauge dependent function.

Acknowledgments

We would like to thank S. S. Gousheh, E. Khajeh and B. Vakili for useful discussions.

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